THE STUDY OF THE EFFICIENCY INDICATORS FOR DIFFERENT TYPES OF PRODUCTION FUNCTIONS

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ABSTRACT: *This study investigates the calculation of the main efficiency indicators for different production functions. The analysis includes the most widely used production functions in economic practice. By using these functions, we provide a comprehensive comparison of their performance in modeling economic production processes. The results of this study will contribute to a better understanding of how different production functions can be used to optimize resources and improve economic efficiency.*

KEY WORDS: *production function, efficiency indicators, CES, Allen, Leontief, quadratic*

JEL CLASSIFICATIONS: *E23, D24*

1. INTRODUCTION

In the current economic context, efficiency and productivity are essential for the growth and sustainable development of economies. Production functions are fundamental tools in economic analysis, providing a theoretical framework for understanding the relationships between inputs and outputs of various production processes. These functions allow economists and policymakers to evaluate economic performance, optimize resource utilization, and formulate effective economic policies.

Of the multitude of production functions existing in the literature, the Cobb-Douglas, CES (Constant Elasticity of Substitution), Row Sato, Allen or Leontief functions are among the most widely used functions on which models are applied in economic analysis. Their flexibility and ability to capture the complex relationships between inputs and outputs in economic systems make these functions extremely useful.

The Cobb-Douglas function, known for its simplicity and constant elasticities, provides a solid foundation for understanding basic production relationships. The CES

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function introduces flexibility through variable elasticities of substitution, allowing for a more detailed analysis of interactions between inputs. An extension of the CES function is the Row Sato function. It provides greater adaptability in capturing complex production dynamics. Another production function used to model production processes in which inputs are used in fixed proportions is the Leontief function. This is also known as the fixed-proportion production function or the constant technical coefficients production function, meaning that there is no substitution between inputs; each unit of output requires a fixed amount of each input.

The study of efficiency indicators is crucial to understanding how different production functions reflect economic realities. Productivity indicators are essential for analyzing economic performance at the micro and macroeconomic levels.

Efficiency indicators:

- ✓ help to assess the economic performance of firms and economies. They allow the identification of sectors or activities that are efficient and those that require improvement;
- ✓ help managers optimize the use of resources. This leads to cost reduction and increased profitability;
- ✓ are essential for the formulation of economic policies. Governments and regulatory institutions can use this information to develop policies that stimulate economic growth and improve competitiveness;
- ✓ allow the monitoring of technological progress and its impact on productivity. This helps to identify technological innovations that contribute most to economic growth;
- ✓ identify inefficiencies in production processes and thus corrective measures can be implemented to improve performance;
- ✓ are used in economic planning and projections to estimate future resource needs and to develop sustainable growth strategies.

This study aims to investigate and compare the efficiency indicators for these production functions, providing a comprehensive perspective on their performances in modeling economic processes. By calculating and interpreting these efficiency indicators for each production function, we will highlight the strengths and limitations of each model.

2. LITERATURE REVIEW

In the economic literature, production functions are essential for understanding the relationships between inputs and outputs in production processes. These production functions have been developed over time by various researchers precisely to meet the needs of economic modeling and to provide more precise and flexible tools for analyzing production processes.

Sudhanshu K. Mishra's 2007 paper provides a detailed analysis of the evolution of the concept of the production function, a central tool of neoclassical economics. The author explores forms of the Cobb-Douglas production function, formulations of the CES function, and how elasticity of substitution became a variable. The paper also discusses the Row Sato function and the biased technical changes incorporated in it (Mishra, 2007). The paper is divided into three main sections: production functions for a single product, production functions for common products, and aggregate production functions. It also addresses the capital controversy and its impact on the economy. It is a valuable resource for understanding the evolution of production functions and their applications in economic analysis.

Many economists consider Philip Wicksteed (1894) to be the first economist to formulate the relationship between output and a series of inputs (factors). However, there is some evidence to suggest that Johann von Thünen first formulated such a function in the 1840s (Humphrey, 1997).

The Cobb-Douglas function, introduced by Charles Cobb and Paul Douglas in the 1920s, is one of the most widely used production functions due to its simplicity and constant elasticities. Somewhat later, in 1928, Cobb and Douglas published a study in which they modeled the growth of the American economy during the period 1899-1922 (Coob & Douglas, 1928). They considered a simplified view of the economy in which production capacity is determined by the amount of labor involved and the value of capital invested. Although there are many other factors that affect economic performance, their model has proven to be an extremely accurate one. At the Romanian level, in the article (Stoicuta, et al., 2016), a comparative study was conducted by estimating the Cobb-Douglas function of real gross value added.

The CES (Constant Elasticity of Substitution) production function, developed by Kenneth Arrow, Hollis Chenery, Bagicha Minhas and Robert Solow in the 1960s, allows for a variable elasticity of substitution between inputs, offering greater flexibility than the Cobb-Douglas function (Minsol, 1968). A comparative study that performs econometric analysis at the macroeconomic level in Romania, by estimating the CES production function is described in the article (Stoicuța & Stoicuța, 2017).

The Row Sato function, developed by Japanese economist Ryuzo Sato, is an extension of the CES function and provides increased flexibility in modeling substitution between inputs (Sato, 1970). It was created to extend the CES (Constant Elasticity of Substitution) function and allow for more flexible modeling of substitution between inputs, thus capturing more complex relationships between factors of production.

The Allen production function, also known as the Allen-Uzawa production function, is a production function that allows for variable elasticity of substitution between inputs (Uzawa, 1962). It is used to model production relations in which the elasticity of substitution is not constant and can vary with input levels.

The Leontief production function, developed by Wassily Leontief in 1951, is used to model production processes in which inputs are used in fixed proportions (Leontief, 1951). It is particularly useful in economic planning studies and in analyzing the impact of economic policies on different sectors.

The quadratic production function is used in economics to model production relationships and has been developed and used by various economists and researchers over time. It is an extension of simpler production functions, such as the Cobb-Douglas function, and allows for greater flexibility in modeling complex relationships between inputs and outputs.

There are also works in the modern specialized literature that offer new perspectives and methods for the analysis of economic productivity and efficiency. Among them, we mention the production function proposed by Ackerberg-Caves-Frazer (ACF) (Ackerberg, et. al., 2015), which addresses problems of endogeneity and functional dependence that can arise in traditional methods of estimating production functions, such as those proposed by (Olley & Pakes, 1996) and (Levinsohn & Petrin, 2003). The Hsieh-Klenow production function is used to analyze productivity and economic growth in emerging economies. This function has been updated and extended in their recent works to include new variables and estimation methods (Hsieh & Klenow, 2009).

The Loecker-Warzynski production function focuses on estimating productivity and profit margins in competitive industries. This function is used to analyze the impact of competition on productivity (Loecker & Warzynski, 2012). Another production function is the one proposed by Gandhi-Navarro-Rivers and focuses on estimating productivity and technological progress in manufacturing industries. This function also uses advanced econometric methods to address endogeneity issues. (Gandhi, et al., 2020).

3. EXAMPLES OF PRODUCTION FUNCTIONS

In this section, we will explore some of the most important production functions used in economic analysis, highlighting their analytical expressions. We have chosen for example production functions that depend on two factors of production (capital and labor).

- *Cobb-Douglas production function (CD):*

$$Q(K,L) = aK^{\alpha}L^{\beta} \tag{1}$$

where a is a scale parameter; α, β there are distribution parameters, K is the capital, L is the labor.

- *CES production function* (Constant Elasticity of Substition)

$$Q(K,L) = a\left[\alpha K^{\varsigma} + (1-\alpha)L^{\varsigma}\right]^{\frac{1}{\xi}} \tag{2}$$

where a is a scale parameter; α is the distribution parameters, and ξ is the parameter that expresses the value of the elasticity of substitution.

- *Row-Sato production function:*

$$Q(K,L) = a\left[\alpha K^{-\varsigma} + (1-\alpha)L^{-\varsigma}\right]^{-\frac{1}{\xi}} \tag{3}$$

where a, b and c are the parameters of the production function.

- *Allen production function:*

$$Q(K,L) = \sqrt{2\gamma KL - \alpha K^2 - \beta L^2} \tag{4}$$

where α, β are distribution parameters, γ is a substitution parameter.

• *Quadratic production function*

$$Q(K,L) = aK^2 + bL^2 + cKL + dK + eL + f \tag{5}$$

where a, b, c, d, e, f are the parameters of the function.

• *Leontief production function:*

$$Q(K,L) = min\left(\frac{K}{a}, \frac{L}{b}\right) \tag{6}$$

4. EFFICIENCY INDICATORS

For a more thorough understanding, we will use the formulations of efficiency indicators for production functions that depend on only two factors: capital and labor. In economic practice, there are three categories of such indicators:

- **Average indicators**

✓ *average productivity in relation to capital, respectively in relation to labor, measures the total output obtained per unit of capital used, respectively per unit of labor used:*

$$W_K = \frac{Q(K,L)}{K},\ W_L = \frac{Q(K,L)}{L} \tag{7}$$

- **Marginal indicators**

✓ *the marginal productivity of capital, respectively of labor, measures the increase in total output obtained by adding an additional unit of capital, keeping labor constant, respectively by adding an additional unit of labor, keeping capital constant:*

$$W_K^{mg} = \frac{\partial Q(K,L)}{\partial K},\ W_L^{mg} = \frac{\partial Q(K,L)}{\partial L} \tag{8}$$

✓ *the marginal rate of substitution of capital by labor measures the amount of capital that must be substituted to add an additional unit of labor, keeping the level of output constant, respectively the marginal rate of substitution of labor by capital measures the amount of labor that must be substituted to add an additional unit of capital, keeping the level of output constant:*

$$R_{K/L}^{mg} = \frac{W_L^{mg}}{W_K^{mg}},\ R_{L/K}^{mgs} = \frac{W_K^{mg}}{W_L^{mg}} \tag{9}$$

- **Elasticity indicators**

✓ *the elasticity of output with respect to capital measures the percentage change in total output in response to a percentage change in capital, holding labor constant, and the elasticity of output with respect to labor measures the percentage change in total output in response to a percentage change in labor, holding capital constant:*

$$E_{Q/K} = \frac{W_K^{mg}}{W_K}\ ,\ E_{Q/L} = \frac{W_L^{mg}}{W_L} \tag{10}$$

The following table presents the expressions of the main efficiency indicators introduced in the above relations, calculated for the production functions introduced above.

Table 1. Calculation of efficiency indicators for the analyzed production functions

No.	Function	Average productivity	Marginal productivities
1	Funcția CD	$W_K = aK^{\alpha-1}L^{\beta}; W_L = aK^{\alpha}L^{\beta-1}$	$W_K^{mg} == \alpha aK^{\alpha-1}L^{\beta}; W_L^{mg} = \beta aK^{\alpha}L^{\beta-1}$
2	Row Sato	$W_K = a\left(\alpha K^{-\xi} + (1-\alpha)L^{-\xi}\right)^{-\frac{1}{\xi}} \Big/ K$ $W_L = a\left(\alpha K^{-\xi} + (1-\alpha)L^{-\xi}\right)^{-\frac{1}{\xi}} \Big/ L$	$W_K^{mg} = a\alpha(\alpha K^{-\xi} + (1-\alpha)L^{-\xi})^{-\frac{1}{\xi}-1} K^{-\xi-1}$ $W_L^{mg} = a(1-\alpha)(\alpha K^{-\xi} + (1-\alpha)L^{-\xi})^{-\frac{1}{\xi}-1} L^{-\xi-1}$
3	CES	$W_K = a\left(\alpha K^{\xi} + (1-\alpha)L^{\xi}\right)^{\frac{1}{\xi}} \Big/ K$ $W_L = a\left(\alpha K^{\xi} + (1-\alpha)L^{\xi}\right)^{\frac{1}{\xi}} \Big/ L$	$W_K^{mg} = a\alpha(\alpha K^{\xi} + (1-\alpha)L^{\xi})^{\frac{1}{\xi}-1} K^{\xi-1}$ $W_L^{mg} = a(1-\alpha)(\alpha K^{\xi} + (1-\alpha)L^{\xi})^{\frac{1}{\xi}-1} L^{\xi-1}$
4	Quadratic	$W_K = \left(aK^2 + bL^2 + cKL + dK + eL + f\right) \Big/ K$ $W_L = \left(aK^2 + bL^2 + cKL + dK + eL + f\right) \Big/ L$	$W_K^{mg} = 2aK + cL + d$ $W_L^{mg} = 2bL + cK + e$
5	Allen	$W_K = \dfrac{\sqrt{2\gamma KL - \alpha K^2 - \beta L^2}}{K}$ $W_L = \dfrac{\sqrt{2\gamma KL - \alpha K^2 - \beta L^2}}{L}$	$W_K^{mg} = \dfrac{\gamma L - \alpha K}{\sqrt{2\gamma KL - \alpha K^2 - \beta L^2}}$ $W_L^{mg} = \dfrac{\gamma K - \beta L}{\sqrt{2\gamma KL - \alpha K^2 - \beta L^2}}$
6	Leontief	$W_K = \dfrac{min\left(\frac{K}{a}, \frac{L}{b}\right)}{K}; W_L = \dfrac{min\left(\frac{K}{a}, \frac{L}{b}\right)}{L}$	$W_K^{mg} = 0 \quad sau \quad \infty; W_L^{mg} = 0 \quad sau \quad \infty$

No.	Production funct.	Marginal rates of substitution	Elasticities
1	CD	$R^{mg}_{K/L} = \frac{\beta a K^{\alpha} L^{\beta-1}}{\alpha a K^{\alpha-1} L^{\beta}} = \frac{\beta}{\alpha}\frac{L}{K}$ $R^{mg}_{L/K} = \frac{\alpha a K^{\alpha-1} L^{\beta}}{\beta a K^{\alpha} L^{\beta-1}} = \frac{\alpha}{\beta}\frac{K}{L}$	$E_{Q/K} = \alpha;\ E_{Q/L} = \beta$
2	Row Sato	$R^{mg}_{K/L} = \frac{(1-\alpha)L^{-\xi-1}}{\alpha K^{-\xi-1}}$ $R^{mg}_{L/K} = \frac{\alpha K^{-\xi-1}}{(1-\alpha)L^{-\xi-1}}$	$E_{Q/K} = \alpha\left(\frac{\alpha K^{-\xi} + (1-\alpha)L^{-\xi}}{K^{-\xi}}\right)^{\frac{-1}{\xi}-1}$ $E_{Q/L} = (1-\alpha)\left(\frac{\alpha K^{-\xi} + (1-\alpha)L^{-\xi}}{L^{-\xi}}\right)^{\frac{-1}{\xi}-1}$
3	CES	$R^{mg}_{K/L} = \frac{(1-\alpha)L^{\xi-1}}{\alpha K^{\xi-1}}$ $R^{mg}_{L/K} = \frac{\alpha K^{\xi-1}}{(1-\alpha)L^{\xi-1}}$	$E_{Q/K} = \alpha\left(\frac{\alpha K^{\xi} + (1-\alpha)L^{\xi}}{K^{\xi}}\right)^{\frac{1}{\xi}-1}$ $E_{Q/L} = (1-\alpha)\left(\frac{\alpha K^{\xi} + (1-\alpha)L^{\xi}}{L^{\xi}}\right)^{\frac{1}{\xi}-1}$
4	Quadratic	$R^{mg}_{K/L} = \frac{2bL + cK + e}{2aK + cL + d}$ $R^{mg}_{L/K} = \frac{2aK + cL + d}{2bL + cK + e}$	$E_{Q/K} = \frac{(2aK + cL + d)\cdot K}{aK^2 + bL^2 + cKL + dK + eL + f}$ $E_{Q/L} = \frac{(2bL + cK + e)\cdot L}{aK^2 + bL^2 + cKL + dK + eL + f}$
5	Allen	$R^{mg}_{K/L} = \frac{\gamma K - \beta L}{\gamma L - \alpha K}$ $R^{mg}_{L/K} = \frac{\gamma L - \alpha K}{\gamma K - \beta L}$	$E_{Q/K} = \frac{(\gamma L - \alpha K)K}{2\gamma KL - \alpha K^2 - \beta L^2}$ $E_{Q/L} = \frac{(\gamma K - \beta L)L}{2\gamma KL - \alpha K^2 - \beta L^2}$
6	Leontief	$R^{mg}_{K/L} = 0\ sau\ \infty;\ R^{mg}_{L/K} = 0\ sau\ \infty$	$E_{Q/K} = E_{Q/L} = 0$

Previous studies have highlighted the importance of using different production functions to model complex relationships between inputs and outputs. The Cobb-Douglas function is appreciated for its simplicity, while the CES and Row Sato functions offer increased flexibility in capturing production dynamics. The Allen-Uzawa function adds an additional level of flexibility through variable elasticity of substitution.

5. DISCUSSIONS

These production functions and the associated efficiency indicators provide economists and decision makers with valuable tools for optimizing resources and improving economic efficiency.

The comparative study of these functions can contribute to a better understanding of how different models can be applied in various economic contexts to maximize productivity and efficiency.

Regarding the comparative analysis of the relationships that represent productivity (efficiency) indicators, we can draw the following specifics:

Table 2. Economic interpretations of the analyzed efficiency indicators

No.	Production funct.	Average productivity	Marginal productivities
1.	CD	If α is large, capital contributes significantly to production. If β is large, labor contributes significantly to production.	If α is large, capital has a significant impact on output. If β is large, labor has a significant impact on output
2.	Row Sato	If α is large, capital contributes significantly to production. If 1−α is large, labor contributes significantly to production.	If α is large, capital has a significant impact on output. If 1−α is large, labor has a significant impact on output.
3.	CES	If α is large, capital contributes significantly to production. If 1−α is large, labor contributes significantly to production.	If α is large, capital has a significant impact on output. If 1−α is large, labor has a significant impact on output.
4.	Quadratic	If a is large, capital has *a* significant impact on output, and the average productivity of capital will increase rapidly with increasing capital. If *d* is large, capital contributes directly to output, and the average productivity of capital will be higher for small values of capital. If *b* is large, labor has a significant impact on output, and the average productivity of labor will increase rapidly with increasing labor. If *e* is large, labor contributes directly to output, and the average productivity of labor will be higher for small values of labor.	If **a** is large, adding capital will have a significant impact on output. If *c* is large, the interaction between capital and labor will influence the marginal productivity of capital. If *d* is large, capital contributes directly to output, and the marginal productivity of capital will be higher for small values of capital. If *b* is large, adding labor will have a significant impact on output. If *c* is large, the interaction between capital and labor will influence the marginal productivity of labor. If *e* is large, labor contributes directly to output, and the marginal productivity of labor will be higher for small values of labor.
5.	Allen	If γ is large, the interaction between capital and labor has a significant impact on output. If α is large, capital contributes negatively to output, reducing the average productivity of capital.	If γ is large, adding capital has a significant positive impact on output. If α is large, adding capital has a negative impact on output. If β is large, adding labor has a

		If β is large, labor contributes negatively to output, reducing the average productivity of labor.	negative impact on output.
6.	Leontief	In the Leontief function, this depends on the coefficient *a* and the amount of labor available. If L is limiting, the average productivity relative to capital will be lower. If K is limiting, the average productivity relative to labor will be lower.	The marginal productivity of capital is zero when capital is not the limiting factor. Adding capital will not increase output if labor is limiting. The marginal productivity of labor is zero when labor is not the limiting factor. Adding labor will not increase output if capital is limiting.

No.	**Production funct.**	**Marginal rates of substitution**	**Elasticities**
1.	CD	If β\α is large, labor can substitute for capital more easily. If α\β is large, capital can substitute for labor more easily.	If α is large, output is very sensitive to changes in capital. If β is large, output is very sensitive to changes in labor.
2.	Row Sato	If (1−α)/α is large, labor can substitute for capital more easily. If α/(1−α) is large, capital can substitute for labor more easily.	If α is large, output is very sensitive to changes in capital. If 1−α is large, output is very sensitive to changes in labor.
3.	CES	If (1−α)/α is large, labor can substitute for capital more easily. If α/(1−α) is large, capital can substitute for labor more easily.	If α is large, output is very sensitive to changes in capital. If 1−α is large, output is very sensitive to changes in labor.
4.	Quadratic	If *b* is large and a is small, labor can substitute for capital more easily, and capital can substitute for labor more easily. If *c* is large, the interaction between capital and labor will influence the marginal rate of substitution.	If *a* is large, output is very sensitive to changes in capital. If *d* is large, capital contributes directly to output, and the elasticity of output with respect to capital will be higher for small values of capital. If *b* is large, output is very sensitive to changes in labor. If *e* is large, labor contributes directly to output, and the elasticity of output with respect to labor will be higher for small values of labor.
5.	Allen	If γ is large and α and β are small, respectively, labor can substitute capital more easily, and capital can substitute labor more easily. If β is large, labor has a negative impact on the substitution of capital.	If γ is large, output is very sensitive to changes in capital and labor, respectively. If α is large, capital contributes negatively to output, reducing the elasticity of output with respect to capital.

		If α is large, capital has a negative impact on the substitution of labor.	If β is large, labor contributes negatively to output, reducing the elasticity of output with respect to labor.
6.	Leontief	Marginal rates of substitution are either 0 or infinite because inputs are used in fixed proportions. There is no substitutability between capital and labor.	Production is not sensitive to changes in capital or labor when capital or labor are not limiting factors.

Each production function has specific advantages and disadvantages, and the choice of the most appropriate one depends on the context and objectives of the economic analysis (Table 3).

Table 3. Performance analysis for the analyzed production functions

No.	Functions	Advantages	Disadvantage
1.	CD	-it is easy to use and interpret. -the elasticities of production with respect to capital and labor are constant. -widely used in economic models due to its simplicity and flexibility.	-does not allow variable substitution between inputs. -cannot capture complex relationships between inputs.
2.	Row-Sato	-allows for variable substitution between inputs. -the elasticity of substitution can be adjusted to reflect different relationships between inputs.	-more complex than the Cobb Douglas function. -requires the estimation of more parameters.
3.	Funcția CES	-allows modeling of substitution between inputs in a more flexible way than CES. -can capture more complex relationships between inputs.	-it is complex and difficult to estimate. -it requires detailed and precise data for estimation.
4.	Quadratic	-allows modeling of complex relationships between inputs and outputs, including higher-order terms. -can capture increasing or decreasing scale effects, depending on the values of the coefficients.	-it is complex, which can make analysis and interpretation of results more difficult. -it may not be suitable for all types of production processes.
5.	Allen	-allows modeling of production relations with variable elasticity of substitution. -can capture complex relationships between inputs.	-is more complex and more difficult to estimate. -requires detailed and precise data for estimation.

6.	Leontief	-it is easy to use and interpret and is suitable for industries where inputs are used in fixed proportions.	-does not allow substitution between inputs. -cannot capture complex relationships between inputs and outputs.

5. CONCLUSIONS

The study of various production functions, such as Cobb-Douglas, CES, Row Sato, quadratic, Allen or Lontief, provides a comprehensive insight into how different industries and economies can optimize resource utilization. Knowing the efficiency indicators for these functions helps to identify the optimal way to allocate resources, for example to maximize production.

It also allows for the assessment of resource efficiency and the identification of areas for improvement. At the same time, it supports economic and political planning decisions by providing information on the relationships between inputs and outputs.

The research results can be applied in various economic sectors to improve efficiency and productivity, having a direct impact on economic growth and competitiveness. It also contributes to the development of economic models that can be used for economic forecasting and analysis and helps estimate the impact of changes in inputs on total output, essential for economic growth strategies.

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